

Practice Test - Chapter 8

Simplify each expression.

1. $\frac{r^2 + rs}{2r} \div \frac{r + s}{16r^2}$

SOLUTION:

$$\begin{aligned}\frac{r^2 + rs}{2r} \div \frac{r + s}{16r^2} &= \frac{r^2 + rs}{2r} \cdot \frac{16r^2}{r + s} \\ &= \frac{r(r + s)}{2r} \cdot \frac{16r^2}{(r + s)} \\ &= 8r^2\end{aligned}$$

2. $\frac{m^2 - 4}{3m^2} \cdot \frac{6m}{2 - m}$

SOLUTION:

$$\begin{aligned}\frac{m^2 - 4}{3m^2} \cdot \frac{6m}{2 - m} &= \frac{(m + 2)(m - 2)}{3m^2} \cdot \frac{6m}{-(m - 2)} \\ &= \frac{6m(m + 2)(m - 2)}{-3m^2(m - 2)} \\ &= -\frac{2(m + 2)}{m}\end{aligned}$$

3. $\frac{m^2 + m - 6}{n^2 - 9} \div \frac{m - 2}{n + 3}$

SOLUTION:

$$\begin{aligned}\frac{m^2 + m - 6}{n^2 - 9} \div \frac{m - 2}{n + 3} &= \frac{m^2 + m - 6}{n^2 - 9} \cdot \frac{(n + 3)}{(m - 2)} \\ &= \frac{(m + 3)(m - 2)(n + 3)}{(n + 3)(n - 3)(m - 2)} \\ &= \frac{m + 3}{n - 3}\end{aligned}$$

4. $\frac{\frac{x^2 + 4x + 3}{x^2 - 2x - 15}}{\frac{x^2 - 1}{x^2 - x - 20}}$

SOLUTION:

$$\begin{aligned}\frac{\frac{x^2 + 4x + 3}{x^2 - 2x - 15}}{\frac{x^2 - 1}{x^2 - x - 20}} &= \frac{x^2 + 4x + 3}{x^2 - 2x - 15} \cdot \frac{x^2 - x - 20}{x^2 - 1} \\ &= \frac{(x + 3)(x + 1)}{(x - 5)(x + 3)} \cdot \frac{(x - 5)(x + 4)}{(x + 1)(x - 1)} \\ &= \frac{(x + 3)(x + 1)(x - 5)(x + 4)}{(x - 5)(x + 3)(x + 1)(x - 1)} \\ &= \frac{x + 4}{x - 1}\end{aligned}$$

5. $\frac{x + 4}{6x + 3} + \frac{1}{2x + 1}$

SOLUTION:

$$\begin{aligned}\frac{x + 4}{6x + 3} + \frac{1}{2x + 1} &= \frac{x + 4}{3(2x + 1)} + \frac{1}{2x + 1} \\ &= \frac{x + 4}{3(2x + 1)} + \frac{1}{2x + 1} \left(\frac{3}{3}\right) \\ &= \frac{x + 4 + 3}{3(2x + 1)} \\ &= \frac{x + 7}{3(2x + 1)}\end{aligned}$$

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$$6. \frac{x}{x^2-1} - \frac{3}{2x+2}$$

SOLUTION:

$$\begin{aligned} \frac{x}{x^2-1} - \frac{3}{2x+2} &= \frac{x}{(x+1)(x-1)} - \frac{3}{2(x+1)} \\ &= \frac{x}{(x+1)(x-1)} \left(\frac{2}{2}\right) - \frac{3}{2(x+1)} \cdot \frac{(x-1)}{(x-1)} \\ &= \frac{2x-3(x-1)}{2(x+1)(x-1)} \\ &= \frac{2x-3x+3}{3(x+1)(x-1)} \\ &= \frac{-x+3}{3(x+1)(x-1)} \end{aligned}$$

$$7. \frac{1}{y} + \frac{2}{7} - \frac{3}{2y^2}$$

SOLUTION:

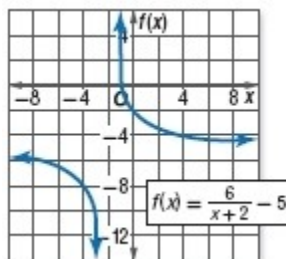
$$\begin{aligned} \frac{1}{y} + \frac{2}{7} - \frac{3}{2y^2} &= \frac{1}{y} \left(\frac{14y}{14y}\right) + \frac{2}{7} \left(\frac{2y^2}{2y^2}\right) - \frac{3}{2y^2} \left(\frac{7}{7}\right) \\ &= \frac{14y}{14y^2} + \frac{4y^2}{14y^2} - \frac{21}{14y^2} \\ &= \frac{4y^2+14y-21}{14y^2} \end{aligned}$$

$$8. \frac{2 + \frac{1}{x}}{5 - \frac{1}{x}}$$

SOLUTION:

$$\begin{aligned} \frac{2 + \frac{1}{x}}{5 - \frac{1}{x}} &= \frac{\left(\frac{2x+1}{x}\right)}{\left(\frac{5x-1}{x}\right)} \\ &= \frac{2x+1}{x} \cdot \left(\frac{x}{5x-1}\right) \\ &= \frac{2x+1}{5x-1} \end{aligned}$$

9. Identify the asymptotes, domain, and range of the function graphed.



SOLUTION:

Vertical asymptote: $x = -2$

Horizontal asymptote: $f(x) = -5$

$$D = \{x \mid x \neq -2\}$$

$$R = \{f(x) \mid f(x) \neq -5\}$$

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10. **MULTIPLE CHOICE** What is the equation for the vertical asymptote of the rational function

$$f(x) = \frac{x+1}{x^2+3x+2}?$$

- A** $x = -2$
B $x = -1$
C $x = 1$
D $x = 2$

SOLUTION:

$$\begin{aligned} f(x) &= \frac{x+1}{x^2+3x+2} \\ &= \frac{x+1}{(x+1)(x+2)} \\ &= \frac{1}{x+2} \end{aligned}$$

Equate the denominator expression to 0 and solve for x .

$$\begin{aligned} x+2 &= 0 \\ x &= -2 \end{aligned}$$

The vertical asymptote of the rational function is $x = -2$.

The correct choice is A.

Graph each function.

11. $f(x) = -\frac{8}{x} - 9$

SOLUTION:

$$\begin{aligned} f(x) &= -\frac{8}{x} - 9 \\ &= \frac{-8-9x}{x} \end{aligned}$$

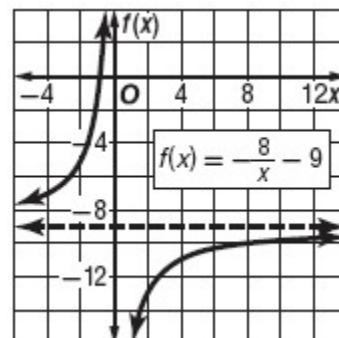
$$\begin{aligned} -8-9x &= 0 \\ 9x &= -8 \\ x &= -\frac{8}{9} \end{aligned}$$

There is a zero at $x = -\frac{8}{9}$. There is a vertical asymptote at $x = 0$.

The degree of the numerator and denominator polynomial is one. Therefore, there is a horizontal asymptote at $y = \frac{-9}{1}$ or $y = -9$.

x	$f(x)$
-2	-5
-1	-1
1	-17
2	-13
4	-11
5	-10.6
8	-10

Draw the asymptotes, and then use the table of values to graph the function.



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12. $f(x) = \frac{2}{x+4}$

SOLUTION:

$$x+4=0$$

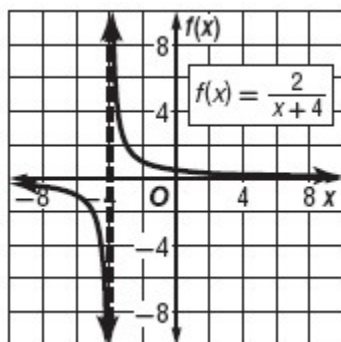
$$x=-4$$

There is a vertical asymptote at $x = -4$.

The degree of the numerator is less than the degree of the denominator, the horizontal asymptote is the line $y = 0$.

x	$f(x)$
-6	-1
-5	-2
-3	2
-2	1
0	0.5
1	0.4

Draw the asymptotes, and then use the table of values to graph the function.



13. $f(x) = \frac{3}{x-1} + 8$

SOLUTION:

$$\begin{aligned} f(x) &= \frac{3}{x-1} + 8 \\ &= \frac{3+8(x-1)}{x-1} \\ &= \frac{3+8x-8}{x-1} \\ &= \frac{8x-5}{x-1} \end{aligned}$$

$$8x-5=0$$

$$8x=5$$

$$x = \frac{5}{8}$$

There is a zero at $x = \frac{5}{8}$.

$$x-1=0$$

$$x=1$$

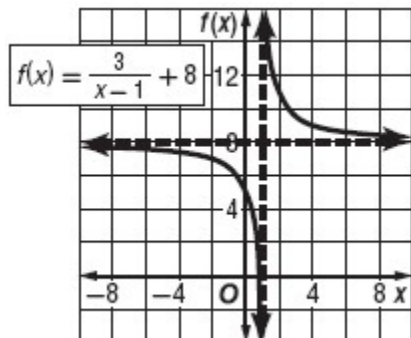
There is a vertical asymptote at $x = 1$.

Since the degree of the numerator and denominator polynomials are equal, there is a horizontal asymptote at $y = 8$.

x	$f(x)$
-5	7.5
-3	7.25
-2	7
-1	6.5
0	5
2	11

Draw the asymptotes, and then use the table of values to graph the function.

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14. $f(x) = \frac{5x}{x+1}$

SOLUTION:

$$5x = 0$$

$$x = 0$$

There is a zero at $x = 0$.

$$x + 1 = 0$$

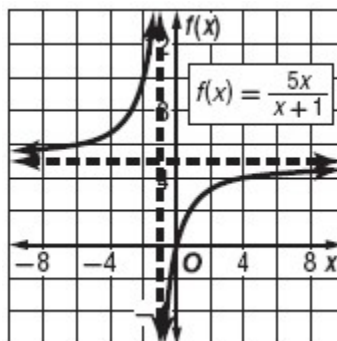
$$x = -1$$

There is a vertical asymptote at $x = -1$.

Since the degree of the numerator and denominator are equal, there is a horizontal asymptote at $y = 5$.

x	$f(x)$
-5	6.25
-3	7.5
-2	10
0	0
1	2.5
4	4

Draw the asymptotes, and then use the table of values to graph the function.



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15. $f(x) = \frac{x}{x-5}$

SOLUTION:

There is a zero at $x = 0$.

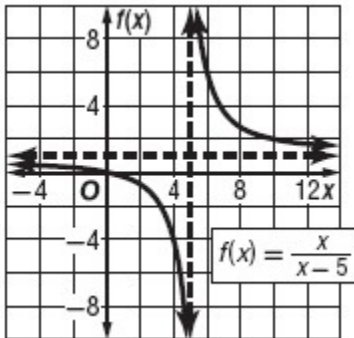
$$\begin{aligned} x - 5 &= 0 \\ x &= 5 \end{aligned}$$

There is a vertical asymptote at $x = 5$.

Since the degree of the numerator and denominator polynomials are equal, there is a horizontal asymptote at $y = 1$.

x	$f(x)$
-5	0.5
0	0
1	-0.25
3	-1.5
4	-4
6	6

Draw the asymptotes, and then use the table of values to graph the function.

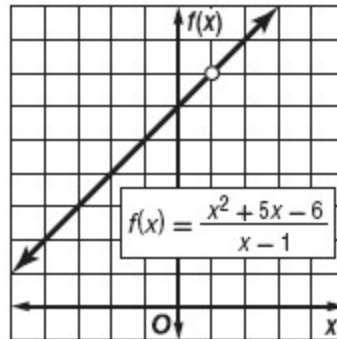


16. $f(x) = \frac{x^2 + 5x - 6}{x - 1}$

SOLUTION:

$$\begin{aligned} f(x) &= \frac{x^2 + 5x - 6}{x - 1} \\ &= \frac{(x + 6)(x - 1)}{x - 1} \\ &= x + 6 \end{aligned}$$

Therefore, the graph of $f(x) = \frac{x^2 + 5x - 6}{x - 1}$ is the graph of $f(x) = x + 6$ with a hole at $x = 1$.



17. Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of the function $f(x) = \frac{x + 5}{x^2 - 2x - 35}$.

SOLUTION:

$$\begin{aligned} f(x) &= \frac{x + 5}{x^2 - 2x - 35} \\ &= \frac{x + 5}{(x - 7)(x + 5)} \\ &= \frac{1}{x - 7} \end{aligned}$$

Therefore, there is a vertical asymptote at $x = 7$ and a hole at $x = -5$.

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18. Determine the equations of any oblique asymptotes in the graph of the function $f(x) = \frac{x^2 + x - 5}{x + 3}$.

SOLUTION:

The difference between the degree of the numerator and the degree of the denominator is 1, so there is an oblique asymptote.

Divide the numerator by the denominator to determine the equation of the oblique asymptote.

$$\begin{aligned} f(x) &= \frac{x^2 + x - 5}{x + 3} \\ &= \frac{(x+3)(x-2)}{(x+3)} \\ &= (x-2) \end{aligned}$$

Therefore, the equation of the oblique asymptote is $f(x) = x - 2$.

Solve each equation or inequality.

19. $\frac{-1}{x+4} = 6 - \frac{x}{x+4}$

SOLUTION:

$$\begin{aligned} \frac{-1}{x+4} &= 6 - \frac{x}{x+4} \\ \frac{-1}{x+4} &= \frac{6(x+4) - x}{x+4} \\ -1 &= 6x + 24 - x \\ -1 &= 5x + 24 \\ 5x &= -25 \\ x &= -5 \end{aligned}$$

Check:

$$\begin{aligned} \frac{-1}{-5+4} &\stackrel{?}{=} 6 - \frac{(-5)}{-5+4} \\ \frac{-1}{-1} &\stackrel{?}{=} 6 + \frac{5}{-1} \\ 1 &\stackrel{?}{=} 6 - 5 \\ 1 &= 1 \checkmark \end{aligned}$$

The solution is $x = -5$.

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20. $\frac{1}{3} = \frac{5}{m+3} + \frac{8}{21}$

SOLUTION:

$$\begin{aligned} \frac{1}{3} &= \frac{5}{m+3} + \frac{8}{21} \\ \frac{1}{3} &= \frac{5(21) + 8(m+3)}{21(m+3)} \\ \frac{1}{3} &= \frac{105 + 8m + 24}{21(m+3)} \\ \frac{1}{3} &= \frac{8m + 129}{21(m+3)} \end{aligned}$$

$$\begin{aligned} 21(m+3) &= 3(8m+129) \\ 21m+63 &= 24m+387 \\ -3m &= 324 \\ m &= -108 \end{aligned}$$

Check:

$$\begin{aligned} \frac{1}{3} &\stackrel{?}{=} \frac{5}{-108+3} + \frac{8}{21} \\ \frac{1}{3} &\stackrel{?}{=} \frac{5}{-105} + \frac{8}{21} \\ \frac{1}{3} &\stackrel{?}{=} -\frac{1}{21} + \frac{8}{21} \\ \frac{1}{3} &\stackrel{?}{=} \frac{-1+8}{21} \\ \frac{1}{3} &\stackrel{?}{=} \frac{7}{21} \\ \frac{1}{3} &= \frac{1}{3} \checkmark \end{aligned}$$

The solution is $m = -108$.

21. $7 + \frac{2}{x} < -\frac{5}{x}$

SOLUTION:

The excluded value for this inequality is $x = 0$.

Solve the related equation $7 + \frac{2}{x} = -\frac{5}{x}$.

$$\begin{aligned} 7 + \frac{2}{x} &= -\frac{5}{x} \\ \frac{7x+2}{x} &= -\frac{5}{x} \\ 7x+2 &= -5 \\ 7x &= -7 \\ x &= -1 \end{aligned}$$

Divide the real line in to three intervals as shown.



Test $x = -2$.

$$\begin{aligned} 7 + \frac{2}{(-2)} &\stackrel{?}{<} -\frac{5}{(-2)} \\ 7 - 1 &\stackrel{?}{<} \frac{5}{2} \\ 6 &\not< \frac{5}{2} \end{aligned}$$

Test $x = -\frac{1}{2}$.

$$\begin{aligned} 7 + \frac{2}{(-\frac{1}{2})} &\stackrel{?}{<} -\frac{5}{(-\frac{1}{2})} \\ 7 - 4 &\stackrel{?}{<} 10 \\ 3 &< 10 \checkmark \end{aligned}$$

Test $x = 1$.

$$\begin{aligned} 7 + \frac{2}{1} &\stackrel{?}{<} -\frac{5}{1} \\ 9 &\not< -5 \end{aligned}$$

The solution of the inequality is $-1 < x < 0$.

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$$22. r + \frac{6}{r} - 5 = 0$$

SOLUTION:

$$r + \frac{6}{r} - 5 = 0$$

$$\frac{r^2 + 6 - 5r}{r} = 0$$

$$r^2 - 5r + 6 = 0$$

$$(r - 3)(r - 2) = 0$$

$$r = 3 \text{ or } r = 2$$

Check: $r = 3$

$$3 + \frac{6}{3} - 5 = 0$$

$$3 + 2 - 5 = 0$$

$$5 - 5 = 0$$

$$0 = 0 \checkmark$$

Check: $r = 2$

$$2 + \frac{6}{2} - 5 = 0$$

$$2 + 3 - 5 = 0$$

$$5 - 5 = 0$$

$$0 = 0 \checkmark$$

$$23. \frac{6}{7} - \frac{3m}{2m-1} = \frac{11}{7}$$

SOLUTION:

$$\frac{6}{7} - \frac{3m}{2m-1} = \frac{11}{7}$$

$$\frac{6(2m-1) - 3m(7)}{7(2m-1)} = \frac{11}{7}$$

$$\frac{12m - 6 - 21m}{14m - 7} = \frac{11}{7}$$

$$\frac{-9m - 6}{14m - 7} = \frac{11}{7}$$

$$7(-9m - 6) = 11(14m - 7)$$

$$-63m - 42 = 154m - 77$$

$$-63m - 154m = -77 + 42$$

$$-217m = -35$$

$$m = \frac{35}{217}$$

$$m = \frac{5}{31}$$

Check: $m = \frac{5}{31}$

$$\frac{6}{7} - \frac{3\left(\frac{5}{31}\right)}{2\left(\frac{5}{31}\right) - 1} = \frac{11}{7}$$

$$\frac{6}{7} - \frac{\left(\frac{15}{31}\right)}{\left(\frac{10-31}{31}\right)} = \frac{11}{7}$$

$$\frac{6}{7} + \frac{15}{21} = \frac{11}{7}$$

$$\frac{18+15}{21} = \frac{11}{7}$$

$$\frac{33}{21} = \frac{11}{7}$$

$$\frac{11}{7} = \frac{11}{7} \checkmark$$

The solution is $m = \frac{5}{31}$.

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24. $\frac{r+2}{3r} = \frac{r+4}{r-2} - \frac{2}{3}$

SOLUTION:

$$\begin{aligned} \frac{r+2}{3r} &= \frac{r+4}{r-2} - \frac{2}{3} \\ \frac{r+2}{3r} &= \frac{3(r+4) - 2(r-2)}{3(r-2)} \\ \frac{r+2}{3r} &= \frac{3r+12-2r+4}{3(r-2)} \end{aligned}$$

$$\begin{aligned} (r+2)(r-2) &= r^2 + 16r \\ r^2 - 4 &= r^2 + 16r \\ r &= -\frac{4}{16} \\ r &= -\frac{1}{4} \end{aligned}$$

Check: $r = -\frac{1}{4}$

$$\begin{aligned} \frac{\left(-\frac{1}{4}\right) + 2}{3\left(-\frac{1}{4}\right)} &? \frac{\left(-\frac{1}{4}\right) + 4}{\left(-\frac{1}{4}\right) - 2} - \frac{2}{3} \\ \frac{\left(\frac{7}{4}\right)}{\left(-\frac{3}{4}\right)} &? \frac{\left(\frac{15}{4}\right)}{\left(-\frac{9}{4}\right)} - \frac{2}{3} \\ -\frac{7}{3} &? \frac{15}{9} - \frac{2}{3} \\ -\frac{7}{3} &? \frac{-15-6}{9} \\ -\frac{7}{3} &? \frac{-21}{9} \\ -\frac{7}{3} &= -\frac{7}{3} \checkmark \end{aligned}$$

The solution is $r = -\frac{1}{4}$.

25. If y varies inversely as x and $y = 18$ when $x = -\frac{1}{2}$, find x when $y = -10$.

SOLUTION:

$$\begin{aligned} y &= \frac{k}{x} \\ 18 &= \frac{k}{\left(-\frac{1}{2}\right)} \\ k &= 18\left(-\frac{1}{2}\right) \\ k &= -9 \end{aligned}$$

Therefore, $y = -\frac{9}{x}$.

Substitute $y = -10$ in the equation $y = -\frac{9}{x}$.

$$\begin{aligned} -10 &= -\frac{9}{x} \\ x &= \frac{9}{10} \end{aligned}$$

26. If m varies directly as n and $m = 24$ when $n = -3$, find n when $m = 30$.

SOLUTION:

$$\begin{aligned} m &= kn \\ 24 &= k(-3) \\ k &= -8 \end{aligned}$$

Substitute $m = 30$ in $m = -8n$.

$$\begin{aligned} m &= -8n \\ 30 &= -8n \\ n &= -\frac{30}{8} \\ &= -\frac{15}{4} \end{aligned}$$

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27. Suppose r varies jointly as s and t . If $s = 20$ when $r = 140$ and $t = -5$, find s when $r = 7$ and $t = 2.5$.

SOLUTION:

$$r = kst$$

$$140 = k(20)(-5)$$

$$140 = -100k$$

$$k = -\frac{140}{100}$$

$$= -\frac{7}{5}$$

Substitute $r = 7$, $t = 2.5$ in the relation $r = -\frac{7}{5}st$.

$$7 = -\frac{7}{5}s(2.5)$$

$$7 = -\frac{7}{2}s$$

$$14 = -7s$$

$$s = \frac{14}{-7}$$

$$s = -2$$

28. **BICYCLING** When Susan rides her bike, the distance that she travels varies directly with the amount of time she is biking. Suppose she bikes 50 miles in 2.5 hours. At this rate, how many hours would it take her to bike 80 miles?

SOLUTION:

$$m = kt$$

$$50 = k(2.5)$$

$$k = \frac{50}{2.5}$$

$$k = 20$$

Substitute $m = 80$ in the relation $m = 20t$.

$$80 = 20t$$

$$80 = 20t$$

$$t = 4$$

So, it would take 4 hours to bike 80 miles.

29. **PAINTING** Peter can paint a house in 10 hours. Melanie can paint the same house in 9 hours. How long would it take if they worked together?

SOLUTION:

$$\frac{x}{10} + \frac{x}{9} = 1$$

$$\frac{9x + 10x}{90} = 1$$

$$\frac{19x}{90} = 1$$

$$19x = 90$$

$$x = \frac{90}{19}$$

$$x \approx 4.7$$

It would take about 4.7 hours if they worked together.

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30. **MULTIPLE CHOICE** How many liters of a 25% acid solution must be added to 30 liters of an 80% acid solution to create a 50% acid solution?

F 18

G 30

H 36

J 66

SOLUTION:

$$x \cdot 25\% = 0.25x$$

$$30 \cdot 80\% = 24$$

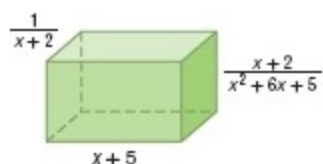
$$(30 + x) \cdot 50\% = 15 + .5x$$

$$24 + 0.25x = 15 + 0.5x$$

$$9 = 0.25x$$

$$36 = x$$

31. What is the volume of the rectangular prism?



SOLUTION:

$$\begin{aligned} \frac{1}{x+2} \cdot \frac{x+2}{x^2+6x+5} \cdot x+5 &= \frac{(x+2)(x+5)}{(x+2)(x^2+6x+5)} \\ &= \frac{(x+2)(x+5)}{(x+2)(x+5)(x+1)} \\ &= \frac{1}{(x+1)} \end{aligned}$$

The volume of the prism is $\frac{1}{x+1}$ cubic units.